Prolegomena to spatial computing : the case of probabilistic cellular automata

Nazim Fatès
Nazim.Fates@loria.fr

LORIA - INRIA - MaIA team

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What about parallel computing ?

Ibn Khaldoun (1332-1406)
Prolegomena / *Muqqadima*

Kant (1724-1804)
Prolegomena to any future metaphysics that will be able to come forward as science

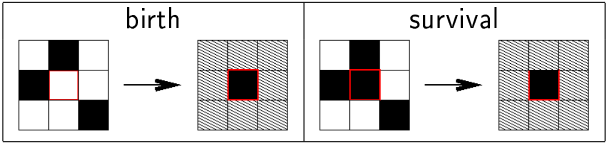
So far, we have followed the path of *sequential* computing
How can we imitate the path followed by the living organisms ?

- **Cellular automata**
  J. von Neumann, S. Ulam, self-reproduction.
- **Partial derivative equations**
  A. Turing, *On chemical basis of morphogenesis* (1952).

First experiment

torical grid, 50×50, two states : $0/1$
8 nearest neighbours
asynchronous updating : α-asynchronous or fully-asynchronous

- demo on the *Game of Life* (Conway, 1970)



- demo on the *majority rule*

Formally speaking...

An **asynchronous** CA is defined as $(Q, \mathcal{L}, V, f, \Delta)$ where :

- Q : set of states
- $\mathcal{L}$: set of cells, grid ($\mathbb{Z}^d$ or subset)
- $V : \mathcal{L} \to \mathcal{L}^k$: neighbourhood, size k.
- $f : Q^k \to Q$: *local* transition rule
- $\Delta : \mathbb{N} \to \mathcal{P}(\mathcal{L})$: updating scheme

The global rule F is : $\begin{array}{lll} F : & Q^{\mathcal{L}} \to & Q^{\mathcal{L}} \\ & (x_c) \to & (x'_c) \end{array}$ where :

$$x'_c = \begin{cases} f(x_{v_1}, x_{v_2}, \ldots, x_{v_k}) & \text{if } c \in \Delta(t) \\ x_c & \text{otherwise} \end{cases}$$

and $(v_1, \ldots, v_k) = V(c)$.

Total number of rules : $\#_{CA} = |Q|^{|Q|^k}$

Exploring a whole space ?

- Dimension 2 :
  For $Q = \{0,1\}$ and $k = 9$, $\#_{CA} = 2^{512} \sim 10^{115}$
- In dimension 1 :
  for $Q = \{0,1\}$ and $k = 3$, $\#_{CA} = 2^8 = 256$
  or 88 equivalent **elementary cellular automata**

Transition table (e.g., majority rule) :

A	B	C	D	E	F	G	H
000	001	100	101	010	011	110	111
0	0	0	1	0	1	1	1

Analysis of asynchronous CA

Restrictions to :

- Elementary cellular automata,
- the two states are quiescent,
- finite size rings of size n.

Fully asynchronous updating scheme :
one cell updated at each time step
Questions :

- Starting from a given configuration, do we converge to a fixed point ?
- If yes, what is the set of reachable fixed points ?
- How much time in average ?

Main result

For the 25 double-quiescent elementary CA :
21 converge a.s., 4 diverge,
the worst expected convergence time in average (WECT) is :

$$0, n \ln n, \Theta\{n^2\}, \Theta\{n^3\}, \Theta\{n.2^n\}, \infty$$

$$WECT = max_{\{x \in Q^{\mathcal{L}}\}} E[T_x]$$

where :
T_x is the convergence time starting from $x \in Q^{\mathcal{L}}$ for $\mathcal{L} = \mathbb{Z}/n\mathbb{Z}$.

Synthetic result

Comportement	ACE (#)	Règle	01	10	010	101	convergence
Identité	204 (1)	$\varnothing$	·	·	·	·	0
Collectionneur de coupons	200 (2)	E	·	·	+	·	$\Theta(n \ln n)$
	232 (1)	DE	·	·	+	+	
Monotone	206 (4)	B	$\leftarrow$	·	·	·	$\Theta(n^2)$
	132 (2)	BC	$\leftarrow$	$\rightarrow$	·	·	
	234 (4)	BDE	$\leftarrow$	·	+	+	
	250 (2)	BCDE	$\leftarrow$	$\rightarrow$	+	+	
	202 (4)	BE	$\leftarrow$	·	+	·	
	192 (4)	EF	$\rightarrow$	·	+	·	
	218 (2)	BCE	$\leftarrow$	$\rightarrow$	+	·	
	128 (2)	EFG	$\rightarrow$	$\leftarrow$	+	·	
Marche aléatoire biaisée	242 (4)	BCDEF	$\leftrightsquigarrow$	$\rightarrow$	+	+	
	130 (4)	BEFG	$\leftrightsquigarrow$	$\leftarrow$	+	·	
Marche aléatoire	226 (2)	BDEF	$\leftrightsquigarrow$	·	+	+	$\Theta(n^3)$
	170 (2)	BDEG	$\leftarrow$	$\leftarrow$	+	+	
	178 (1)	BCDEFG	$\leftrightsquigarrow$	$\leftrightsquigarrow$	+	+	
	194 (4)	BEF	$\leftrightsquigarrow$	·	+	·	
	138 (4)	BEG	$\leftarrow$	$\leftarrow$	+	·	
	146 (2)	BCEFG	$\leftrightsquigarrow$	$\leftrightsquigarrow$	+	·	
Marche aléatoire biaisée	210 (4)	BCEF	$\leftrightsquigarrow$	$\rightarrow$	+	·	$\Omega(n2^n)$
Divergent	198 (2)	BF	$\leftrightsquigarrow$	·	·	·	Divergent
	142 (2)	BG	$\leftarrow$	$\leftarrow$	·	·	
	214 (4)	BCF	$\leftrightsquigarrow$	$\rightarrow$	·	·	
	150 (1)	BCFG	$\leftrightsquigarrow$	$\leftrightsquigarrow$	·	·	

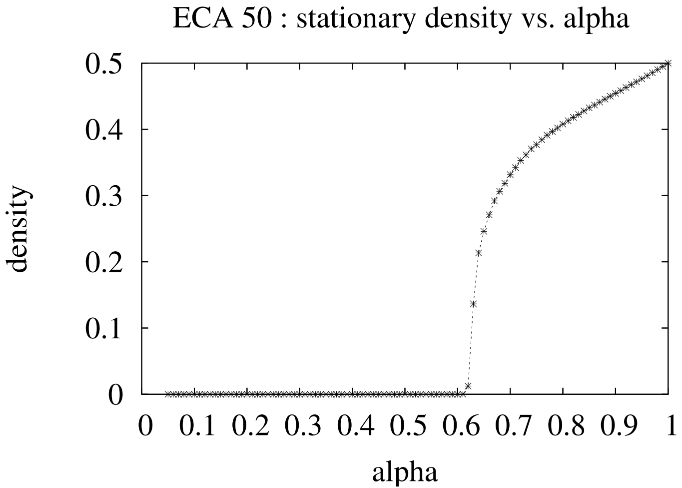
What about the α-asynchronous dynamics ?

Extension of the previous classification :
PhD of Damien Regnault (ENS Lyon)
but brutal changes of behaviours observed !
demo for ECA 50 :

$$\forall (a,b,c) \in \{0,1\}^3,\ f(a,b,c) = \begin{cases} \bar{b} & \text{if } (a,b,c) \neq (0,0,0) \\ 0 & \text{if } (a,b,c) = (0,0,0) \end{cases}$$

Looking at the "asymptotic" density

$n = 10^4$, $T = 10^4$

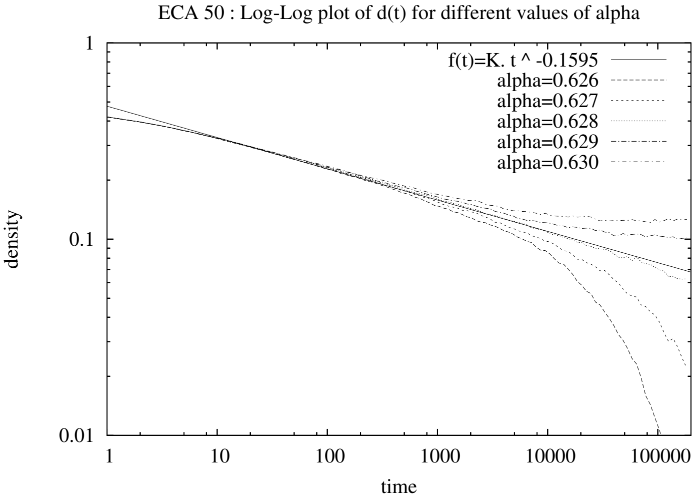


What happens for $\alpha \sim 0.62$?

temporal evolution suggests **directed percolation**

Near criticality

$n = 2 * 10^4$



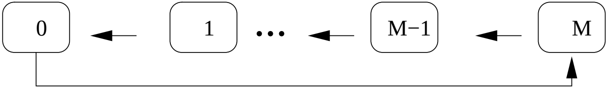
confirms directed percolation hypothesis (seven ECA)

Coupling reaction-diffusion and chemotaxis

The AMIBES project with H. Berry, A. Boyer, V. Chevrier, B. Girau, O. Simonin.

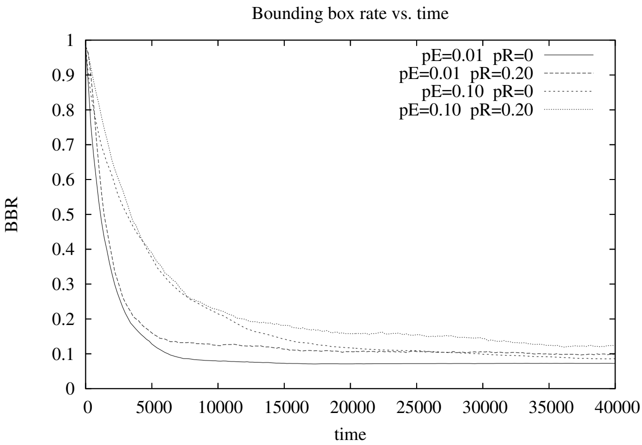
Inspiration from *Dictyostelium discoideum*

- Environment layer :
  0 = neutral, M =excited, 1 … M-1 = refractory



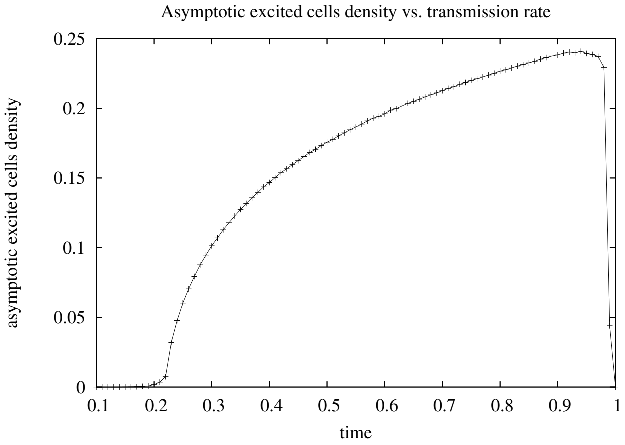
  Excitation happens with probability p_T
- Amoebae layer :
  Normal move = follow excitation (random choice)
  Random move with probability p_R
- Emission of excitations :
  If state is neutral emit with probability p_E

Evolution of BBR



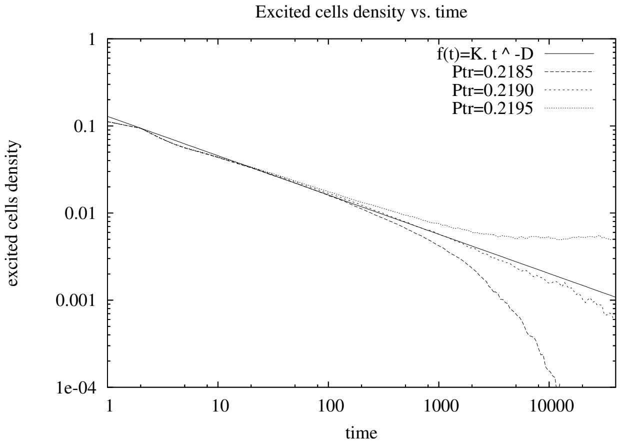
Asymptotic excited cells density

Grid size 100*100



Temporal evolution of excited cells density

Grid size 800*800



Consequences for “spatial” computing ?

- Modelling approach : beware of synchronous models !
- Complexity issue : two states and small neighbourhood are sufficient for observing various phenomena
- Phase transitions are surprising phenomena

These phenomena are not only observed for two-states models but also appear in solving the *decentralised grouping* problem.

These studies may consitute an interesting preparation before realising *amorphous computing*.

"Thank you..."

... to H. Berry, A. Boyer, V. Chevrier, B. Girau, O. Simonin.

... to M. Morvan, N. Schabanel, E. Thierry, D. Regnault

... to you, for your attention !

Zao Wou Ki, 1987, coll. ENS Lyon.